PHY306: Homework#1 Solutions, Spring 10

by TA: Yi Gu

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1 Problem 1.08(2+1+2pts)

(a)Assume $T_{night} = -25^oC$ and $T_{day} = 35^oC$. Then,$\triangle T = 35 - (-25) = 60^oC$. So, variation in length,

$$\triangle L = \alpha L \triangle T = (1.1 \times 10^{-5} K^{-1})(1Km)(60k) = 66CM$$

(b)The metals used have different $\alpha' s$, so variation in temperature makes one metal expand/contract more than the other leading the metal strip to coil or uncoil. This 'temperature-dependent' coiling is calibrated to correspond to the 'numbers' on the dial.

(c)Ignore the edge effects:

$$\begin{aligned} \beta &= \frac{\triangle V}{V \triangle T} \\ &= \frac{(\triangle L_x) L_y L_z + L_x (\triangle L_y) L_z + L_x L_y (\triangle L_z)}{(L_x L_y L_z) \triangle T} \\ &= \frac{\triangle L_x}{L_x \triangle T} + \frac{\triangle L_y}{L_y \triangle T} + \frac{\triangle L_z}{L_z \triangle T} \\ &= \alpha_x + \alpha_y + \alpha_z. \end{aligned} \tag{1}$$

2 Problem 1.10(5pts)

Assume an average-sized room to have dimension $(5 \times 4 \times 3)m^3$ at 1 atm and 300K. Then the number of air molecules,

$$N = \frac{PV}{kT} = \frac{\left(10^5 pa\right)\left(60m^3\right)}{\left(1.38 \times 10^{-23} J/K\right)(300K)} = 1.45 \times 10^{27}. \tag{2}$$

3 Problem 1.16(3+2+3+2pts)

(a)Assume horizontal slab of air(with density)ρ to have cross section area A and mass M. Then, for equilibrium:

$$\begin{aligned} P(z)A &= P(z+dz)A + Mg \\ &\Rightarrow \left(P(z+dz) - P(z)\right)A = -(\rho A dz)g \\ &\Rightarrow \frac{dP}{dz} = -\rho g. \end{aligned} \tag{3}$$

(b)Use ideal gas law:

$$\frac{dP}{dz} = -\frac{M}{V}g = -\frac{Nm}{V}g = -\frac{mg}{kT}P \tag{4}$$

(c)Plug the given solution: $P(z) = P(0)e^{-mgz/kT}$ in the above equation (4) and check if that works. To show that ρ obeys a similar(same!)equation, use the results of (a)and(b):

$$\rho(z) = \tfrac{-1}{g}\left(-\tfrac{mg}{kT}P(0)e^{-mgz/kT}\right) = \rho(0)e^{-mgz/kT}$$

where,$\rho(0) = \frac{mP(0)}{kT}$.

(d)Assume sea level to be at $z = 0$ such that $P(0) = 1$ atm. Molecular mass of air$(0.8N_2+0.2O_2)$, $m = \frac{(0.8\times28)+(0.2\times32}{6.023\times10^{23}} = 4.8\times10^{-26}Kg/mol$. Plugging these values and $T = 280K$ in expression for P(z),Table. 1 is obtained.

4 Problem 1.20(1+2+2pts)

Atomic mass of Fluorine, $M_F = 19g$ and $M_{UF_6} = M_U + 6M_F$. Use these to get:

Location	$Height, z(m)$	$Pressure, P(z) = e^{-(1.2\times10^{-4})z}(atm)$
Ogden,UT	1430	0.84
Leadville,CO	3090	0.69
Mt.Whitney,CA	4420	0.58
Mt.Everest,Nepal	8850	0.34

Table 1: Solution for 1.16(d)

$$\begin{aligned} v_{r,s}\left({}^{238}UF_6\right) &= \sqrt{\frac{3RT}{M_{UF_6}}} = \sqrt{\frac{3(8.314J/K)(300K)}{(0.238+6\times0.019)Kg}} = 145.8m/s \\ v_{r,s}\left({}^{235}UF_6\right) &= \sqrt{\frac{3(8.314J/K)(300K)}{(0.238+6\times0.019)Kg}} = 146.4m/s. \end{aligned} \quad (5)$$

5 Problem 1.21(5pts)

Average force exerted by the hailstones on the window is equal to the rate of change of momentum (its component perpendicular to the window)of the hailstones. So, average pressure,

$$\begin{aligned} \bar{P} &= \frac{\bar{F}}{A} = \frac{m\triangle v_p}{A\triangle_t} = \frac{m(2vcos45^o)}{A\triangle t} \\ &= \frac{(0.002Kg)\left(\sqrt{2}\cdot 15m/s\right)}{(0.5m^2)(1/30s)} = 2.5Pa \sim 2.5\times10^{-5}atm! \end{aligned} \quad (6)$$

6 Problem 1.23(2+3pts)

Helium (monoatomic) has translational d.o.f only $\Rightarrow f = 3$:

$$U = \frac{3}{2}NkT = \frac{3}{2}PV = \frac{3\left(10^5Pa)\right)\left(10^{-3}m^3\right)}{2} = 150J \quad (7)$$

Air $(N_2 + O_2)$ has translational and rotational d.o.f $\Rightarrow f = 3 + 2 = 5$:

$$U = \frac{5}{2}PV = 250J \tag{8}$$

7 Problem 1.3(1+1+1+1+1pt)

(a)(b)The work done is minus the area in the Fig. 1.The easiest way to compute this area is to note that the average pressure during the process is 2atm, so

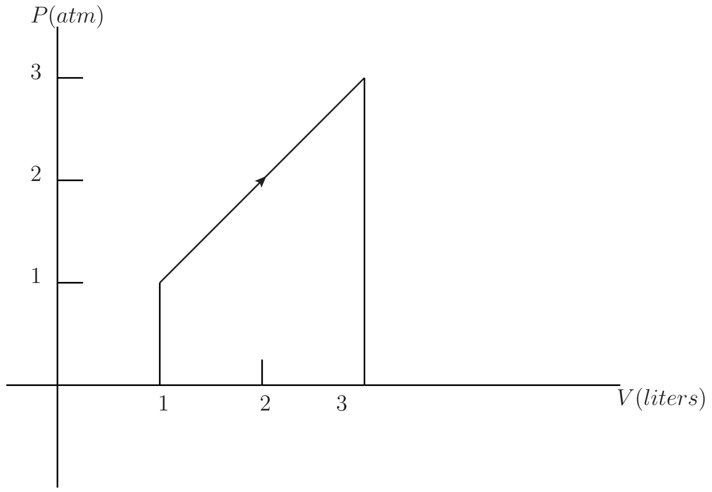

Figure 1: P-V curve for 1.31

$$\begin{aligned} W &= -Area = -\bar{P}\triangle V = -(2atm)(2liters) \\ &= -(2\times 10^5 Pa)(2\times 10^{-3}m^3) = -400J. \end{aligned} \tag{9}$$

The minus sign indicates that 400J of work is done *by* the gas on its surroundings.

(c)Energy change,

$$\triangle U = \frac{3}{2}\left[P_f V_f - P_i V_i\right] = \frac{3}{2}\left[(3)(3) - (1)(1)\right] atm\cdot liter = 1200J. \tag{10}$$

(d)Use first law:

$$\triangle U = Q + W \Rightarrow Q = 1200 - (-400) = 1600J. \tag{11}$$

This amount of heat wad added to He during the expansion process.

(e)Heat the cylinder and let the piston move out such that the P-V curse is followed!

8 1.34(4+2+4pts)

(a)Use $W = -P\triangle V$:

$$\begin{aligned} \text{A:}W &= 0, \\ \text{B:}W &= -P_2(V_2 - V_1), \\ \text{C:}W &= 0, \\ \text{D:}W &= -P_1(V_1 - V_2). \end{aligned}$$

(b)A:Add heat to the gas but keep the piston fixed so that the volume does not change.
B:Let the piston move out but keep adding heat so that the pressure does not change.
C:Hold the piston fixed and cool the gas to reduce the pressure.
D:Push the piston in but keep cooling the gas to maintain the pressure.

(c)Use $\triangle U = \frac{5}{2}\left[P_f V_f - P_i V_i\right]$ and First law to get table. 2 below,

Step	W	$\triangle U$	$Q = \triangle U - W$
A	0	$\frac{5}{2}V_1(P_2 - P_1)$	$\frac{5}{2}V_1(P_2 - P_1)$
B	$-P_2(V_2 - V_1)$	$\frac{5}{2}P_2(V_2 - V_1)$	$\frac{7}{2}P_2(V_2 - V_1)$
C	0	$\frac{5}{2}V_2(P_1 - P_2)$	$\frac{5}{2}V_2(P_1 - P_2)$
D	$P_1(V_2 - V_1)$	$\frac{5}{2}P_1(V_1 - V_2)$	$\frac{7}{2}P_1(V_1 - V_2)$
Entire cycle	$-(P_2 - P_1)(V_2 - V_1)$	0	$(P_2 - P_1)(V_2 - V_1)$

Table 2: Solution for 1.34(c)