PHY306: Homework#5 Solutions, Spring 10

by TA: Yi Gu

Mar. 11, 2010

1 Problem 3.32(10pts)

This is a non-quasistatic compression.
(a)The work I do is the force I exert times the displancement:

$$W = (2000N)(0.001m) = 2J. \tag{1}$$

(b)Because of the assumption that you push the piston in *very suddenly*, you can treat it as adiabatic compression process. Therefore, there is absolutely no heat added to cylinder. There was no spontaneous flow of energy from a hot object to a cold one.
(c)By the first law,

$$\triangle U = Q + W = 0 + 2J = 2J. \tag{2}$$

(d)The change in volume is

$$\triangle V = -(0.01m^2)(0.001m) = -10^{-5}m^3 \tag{3}$$

so,

$$\triangle S = \frac{1}{T}\triangle U + \frac{P}{T}\triangle V = \frac{2J + (10^5 N/m^2)(-10^{-5}m^3)}{300K} = \frac{2J - 1J}{300K} = \frac{1}{300}J/K. \tag{4}$$

At that very moment when I push the cylinder in a very sudden, the compression is adiabatic, and irreversible. However, after that moment, the system experiences reversible and quasistatic process, where the derivative form of 1st law applies.I have created entropy, and the force exerted on the piston from outside was twice as great as the force exerted by the gas from inside.

2 Problem 3.35(5pts)

With three oscillators and four units of energy, the multiplicity is $\begin{pmatrix} 4+3-1 \\ 4 \end{pmatrix} = 15$. If we now add an oscillator without removing any energy, the multiplicity increases to $\begin{pmatrix} 4+4-1 \\ 4 \end{pmatrix} = 35$. If we remove on unit of energy, the multiplicity is then $\begin{pmatrix} 3+4-1 \\ 3 \end{pmatrix} = 20$, still larger than what we started with. If we remove two units of energy, the multiplicity decreases to $\begin{pmatrix} 2+4-1 \\ 2 \end{pmatrix} = 10$, which is too small. So apparently, to hold the multiplicity (and entropy) fixed while adding an oscillator, we need to remove somewhere between one and two units of energy (whatever that means), i.e., μ is somewhere between $-\epsilon$ and -2ϵ.

3 Problem 3.36(10pts)

(a) We start with the entropy computing,

$$\begin{aligned} S &= kln\Omega \\ &= kln\left(\frac{q+N}{q}\right)^q + kln\left(\frac{q+N}{N}\right)^N \\ &= kq \cdot ln\left(\frac{q+N}{q}\right) + kN \cdot ln\left(\frac{q+N}{N}\right) \end{aligned} \tag{5}$$

To compute the chemical potential, we need the derivative

$$\begin{aligned} \frac{\partial S}{\partial N} &= kq\left(\frac{1}{1+N/q}\right)\frac{1}{q} + kln\left(1+\frac{q}{N}\right) + kN\left(\frac{1}{1+q/N}\right)\left(-\frac{q}{N^2}\right) \\ &= k\left(\frac{q}{q+N}\right) + kln\left(1+\frac{q}{N}\right) - k\left(\frac{q}{N+q}\right) \\ &= kln\left(1+\frac{q}{N}\right) \end{aligned} \tag{6}$$

The chemical potential is therefore

$$\mu = -T\frac{\partial S}{\partial N} = -kTln\left(1+\frac{q}{N}\right). \tag{7}$$

(b)In the limit $N \gg q$, the logarithm is approximately q/N, so $\mu \approx -kTq/N$. This says that when we add a "particle" to the system but no energy, the entropy in fundamental units increases by q/N, a number much less than 1. In the other limit, $q \gg N$, the logarithm is approximately $ln(q/N)$, so $\mu \approx -kTln(q/N)$ and therefore, when we add a "particle" to the system but no energy, the entropy in fundamental units increases by $ln(q/N)$, a number somewhat larger than 1. This is significantly larger increase than in the first case. In other words, when there is already a large excess of particles over energy, adding another particle doesnot increase the entropy too much. But when there is an excess of energy units over particles, adding another particle gives a significant increase in entropy. Basically, the system "wants" to gain particles more in the second case than in the first.

4 Problem 4.1(3+2pts)

(a)The net work done by the gas during one cycle is

$$|W| = (P_2 - P_1)(V_2 - V_1) = (P_1)(2V_1) = 2P_1V_1 \tag{8}$$

while the heat absorbed (during steps A and B) is

$$Q_h = \frac{5}{2}V_1(P_2 - P_1) + \frac{7}{2}P_2(V_2 - V_1) = \frac{5}{2}V_1P_1 + 14P_1V_1 = \frac{33}{2}P_1V_1. \tag{9}$$

Therefore the efficiency is

$$\eta = \frac{|W|}{Q_h} = \frac{2P_1V_1}{\frac{33}{2}P_1V_1} = \frac{4}{33} = 12\% \tag{10}$$

(b)The relative temperatures at various points around the cycle can be determined from the ideal gas law, $PV = NkT$. The lowest temperature occurs at the bottom-left corner when P and V are both smallest. As the pressure doubles during step A, the temperature also doubles; then as the volume is tripled during step B, so is the temperature. Thus the highest temperature, at the upper-right corner, is six times as great as the lowest temperature. For these extreme temperatures, the maximum possible efficiency would be

$$\eta_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_c}{6T_c} = \frac{5}{6} = 83\% \tag{11}$$

The rectangular cycle is extremely inefficient compared to a Carnot cycle.

5 Problem 4.3(3+2+5pts)

(a)An efficiency of 40% means that the other 60% of the energy consumed ends up as waste heat. That is 1.5 times as much as the amount that ends up as useful work. More generally, by the definition of efficiency and the first law,

$$\eta = \frac{W}{Q_{hot}} = \frac{W}{Q_{cold} + W}, \tag{12}$$

so the waste heat is

$$Q_c = W\left(\frac{1}{\eta} - 1\right) = 1.5W = 1.5(GW) \tag{13}$$

(b)In one second, the waste heat dumped to the river is 1.5× 10^9J, and this heat is spread among 10^5 kg of water, so each kilogram gets 15kJ. With a heat capacity of $4186J/^oC$, the water's temperature increases by $\triangle T = Q/C$ =$15000J/4186J/^oC$=3.6^oC (c)The latent heat to evaporate water is 2260J/g (at 100^oC). At room temperature it is about 8% more, as mentioned in Problem 1.54 and Figure 5.11; so I will take $L = 2400J/g$. The total amount of water that must evaporate each second is then

$$\frac{1.5 \times 10^9 J}{2400J/g} = 6 \times 10^5 g = 600kg. \tag{14}$$

That is only 0.6 m^3, or only 0.6% of the water in the river.

6 Problem 4.5(10pts)

To compute Q_h and Q_c we need consider only the isothermal process 1-2 and 3-4, since the other two steps are adiabatic. Furthermore, the heat input during an isothermal process is equal in magnitude to the work performed, since for an ideal gas $\triangle U \propto \triangle T$ =0. Therefore, the heat input is

$$Q_h = |W_{12}| = \int_{V_1}^{V_2} PdV = NkT_h ln\frac{V_2}{V_1} \tag{15}$$

and similarly,

$$Q_c = |W_{34}| = \int_{V_4}^{V_3} PdV = NkT_c ln \frac{V_3}{V_4} \tag{16}$$

The efficiency of the engine is

$$\eta = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h} \frac{ln(V_3/V_4)}{ln(V_2/V_1)}, \tag{17}$$

which is equal to the Carnot efficiency provided that V_3/V_4=V_2/V_1.To show that this is true, note from equation 1.39 that for each of the adiabatic processes, $VT^{f/2}$ is constant (where f is the number of degrees of freedom per molecule). For the adiabatic expansion 2-3, this implies that

$$V_3 T_c^{f/2} = V_2 T_h^{f/2} \tag{18}$$

while for the adiabatic compression 4-1 we have

$$V_4 T_c^{f/2} = V_1 T_h^{f/2} \tag{19}$$

Dividing these two equations, we obtain V_3/V_4=V_2/V_1, as needed to cancel the logarithms in the preceding formula for the efficiency