PHY306: Homework#6 Solutions, Spring 10

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1 Problem 4.10(5pts)

As computed in the text, an ideal kitchen refrigerator could have a COP of about

$$COP = \frac{T_c}{T_h - T_c} = \frac{255K}{298K - 255K} = 5.9$$

Therefore, by the definition of COP, Q_c=5.9W or W=Q_c/5.9. In each second, this refrigerator mush remove 300J of heat from the inside, so the work required is W=300J/5.9=50 J. In other words, the power drawn from the wall could be as little as 50 W.

2 Problem 4.12(5pts)

For an ideal gas to execute the rectangular PV cycle shown below (in the Figure. 1),
The engine must absorb heat throughout steps A and B and expel heat throughout steps C and D. The temperature of the gas is proportional to the product PV, so it varies continuously throughout the cycle, increasing during steps A and B and decreasing during steps C and D. But this means that the gas must absorb heat over the full range of temperatures in the cycle, then expel heat over the same full range of temperatures. In a refrigerator, however, the working substance must absorb heat from a source that is significantly colder than the reservoir to which it expels heat. In other words, the range of temperatures over which it absorbs heat must lie entirely below the range of temperatures over which it expels heat. Since the rectangular cycle does not have this property, it cannot function as a working refrigerator.

3 Problem 4.14(2+3+3+2pts)

(a) The COP should be defined as the benefit divided by the cost. In this case the benefit is the heat enters the building, Q_h, while the cost is the electrical energy consumed, W. So benefit/cost = Q_h/W.

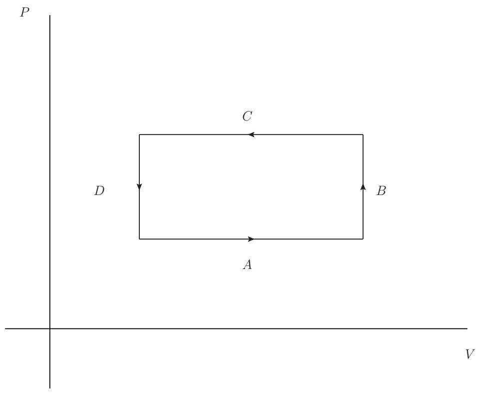

Figure 1: PV diagram for rectangular cycle

(b) The energy in is Q_c+W and the energy out is Q_h, so

$$Q_h = Q_c + W$$

under cyclic operation. The COP is therefore

$$COP = \frac{Q_h}{Q_h - Q_c} = \frac{1}{1 - Q_c/Q_h},$$

which is always greater than 1. (c) The entropy expelled during the cycle must be at least as great as the entropy absorbed, so

$$\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c} i.e. \frac{T_c}{T_h} \geq \frac{Q_c}{Q_h}$$

Because Q_c/Q_h must be less than or equal to T_c/T_h, the quantity 1-Q_c/Q_h must be greater than or equal to 1-T_c/T_h, and therefore, by the result of part (b),

$$COP \leq \frac{1}{1 - T_c/T_h} = \frac{T_h}{T_h - T_c}$$

(d) For an electric heater, all the electrical energy (W) is converted to heat (Q_h),so the COP is 1. An ideal heat pump, though, always has a COP greater than 1. For instance, if T_h=25^{o}C and T_c=0^{o}C, then the COP can (in principle) be as high as 298/25≃12. In practice the COP is never this high, but as long as T_h and T_c are not too different, a heat pump offers a big advantage in efficiency over an electric heater. On the other hand, a heat pump is more expensive to manufacture and maintain, since it a complicated device with many moving parts. Fortunately, a central air conditioning system can double as a heat pump in the winter. So if you are already planning to install central air, and your winters are not too cold, get a heat pump.

4 Problem 4.15(2+3+5pts)

(a) The COP should be defined as the benefit/cost ratio, the quantity that we want to be as large as possible. Here the benefit is Q_c and the cost is Q_f, so COP=Q_c/Q_f. (b) Cyclic operation requires that $\triangle$ U for the working substance be zero. Therefore the total energy entering during a cycle must equal the total energy leaving:

$$Q_f + Q_c = Q_r$$

This relation says nothing about the ratio Q_c/Q_f (either Q_c or Q_f could be bigger than the other), so energy conservation does permit the COP to be greater than 1. (c) For cyclic operation the working substance cannot gain or lose any entropy over the long run, so the second law tells us that the total entropy expelled must be at least as much as the total entropy absorbed:

$$\frac{Q_r}{T_r} \geq \frac{Q_c}{T_c} + \frac{Q_f}{T_f}$$

Since the COP involves Q_c and Q_f but not Q_r,let us use energy conservation to eliminate Q_r:

$$\frac{Q_f}{T_r} + \frac{Q_c}{T_r} \geq \frac{Q_c}{T_c} + \frac{Q_f}{T_f} or \frac{Q_f}{T_r} - \frac{Q_f}{T_f} \geq \frac{Q_c}{T_c} - \frac{Q_c}{T_r}$$

Solving for Q_c/Q_f and being careful with the direction of the inequality, we find

$$\frac{Q_c}{Q_f} \leq \frac{\frac{1}{T_r} - \frac{1}{T_f}}{\frac{1}{T_c} - \frac{1}{T_r}} = \frac{T_c\left(T_f - T_r\right)}{T_f\left(T_r - T_c\right)}$$

This is the desired limit on the COP, in terms of the three temperatures. As expected, high T_f is good,as is a small difference between T_r and T_c.

5 Problem 4.21(3+2+5pts)

(a)The cycle consists of isothermal processes (at T_h and T_c) connected by constant volume processes: (in the Figure. 2),

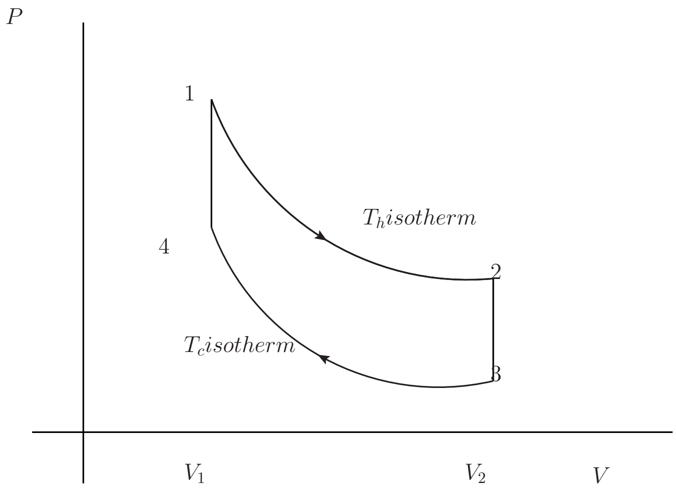

Figure 2: Stirling engine cycle

(b)To calculate the efficiency we need to know the net work done and the total heat input for one cycle. The work done by the gas during the power stroke is:

$$W_{12} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{NkT_h}{V} dV = NkT_h ln\frac{V_2}{V_1}$$

and the work done by the gas during the compression stroke is similarly

$$W_{34} = \int_{V_2}^{V_1} PdV = \int_{V_1}^{V_2} \frac{NkT_c}{V} dV = -NkT_c ln\frac{V_2}{V_1}$$

so the net work done per cycle is

$$W = W_{12} + W_{34} = Nk(T_h - T_c) ln\frac{V_2}{V_1}$$

Meanwhile the heat input occurs during the power stroke and during the transfer to the hot cylinder. Because the power stroke is isothermal, the energy of the gas doesn't change during this step and therefore, by the first law,

$$Q_{12} = W_{12} = NkT_h ln\frac{V_2}{V_1}$$

During the transfer to the hot cylinder, there is no work done, so by the first law and the equipartition theorem, the heat input is

$$Q_{41} = U_1 - U_4 = \frac{f}{2}Nk\left(T_h - T_c\right),$$

where f is the number of degrees of freedom per molecule. Thus the total heat input is

$$Q_h = Q_{12} + Q_{41} = NkT_h ln\frac{V_2}{V_1} + \frac{f}{2}Nk\left(T_h - T_c\right).$$

Now the efficiency,e, is defined as e=W/Q_h.

$$e = \frac{W}{Q_h} = \frac{Nk\left(T_h - T_c\right) ln(V_2/V_1)}{NkT_h ln(V_2/V_1) + \frac{f}{2}Nk\left(T_h - T_c\right)}$$

It is just algebraically simpler to compute the reciprocal to the efficiency next,

$$\frac{1}{e} = \frac{T_h}{T_h - T_c} + \frac{f}{2ln(V_2/V_1)}$$

The first term is just the reciprocal of the Carnot efficiency, e_C=1-(T_c/T_h), so we can write,

$$\frac{1}{e} = \frac{1}{e_C} + \frac{f}{2ln(V_2/V_1)}$$

The second term is always positive, but is smaller for large compression ratios. Therefore 1/e is always greater than 1/e_C, that is, e is less than e_C, by an amount that is smaller when the compression ratio is large. For instance, if T_h is twice as large as T_c (say 600K compared to 300K), so e_C =1/2, and if the compression ratio is 10 and the gas have five degrees of freedom per molecule, then

$$\frac{1}{e} = \frac{1}{1/2} + \frac{5}{2ln10} = 2 + \frac{5}{2 \times 2.3} = 3.1$$

That is efficiency is about 32%.
(c)With an ideal regenerator, the hat input during step 41 comes for free, because it is exactly the same($\frac{f}{2}$Nk(T_h-T_c)) as the heat output during step 23. Therefore only Q_{12} should be counted as part of Q_h when computing the efficiency. Following the steps of the preceding calculation, this means that the second term in the expression for 1/e is no longer present, and therefore, e=e_C.
(d)To really obtain e=e_C, the temperature difference between the gas and whichever reservoir it is exchanging heat with would always have to be infinitesimal; therefore the engine would have to operate infinitely slowly, just like an ideal Carnot engine. To get any power out of a Stirling engine you have to run it with nonzero temperature differences. You might think, then, that there is no advantage to a Stirling engine over a Carnot engine, but in fact the Stirling engine turns out to be easier (mechanically) to build and operate, since the walls the cylinders can always be at the same temperature, and are always in approximate thermal equilibrium with the gas that is near them. Both the Stirling and Carnot engines should be contrasted with internal combustion engines, which are generally more compact (no external combustion chamber) but are limited to certain types of fuel and are more polluting because the fuel never burns completely. Compared to a steam engine, a Stirling engine has the advantage of greater simplicity and possibly higher efficiency, but the disadvantage of (probably) lower power. At present, it seems that some other type of engine is considered more practical than the Stirling engine for virtually every application. However, a Stirling engine can be operated in reverse as a refrigerator, and I have read that Stirling refrigerators are quite practical for liquefying small amounts of air.

6 Problem 4.33(0pts)

Since most of you get confused on this problem, I will not count this in!
(a)The initial (molar) enthalpy is 8174J. You start with enthalpy at 100bar pressure and 300K, which corresponds to 8174 J/mol. Since the process ends up at 1 bar pressure, to keep the molar enthalpy fixed, the temperature must decrease to some point between 200K to 300K. You just use (8717-5800)J/100K to calculate change rate of molar enthalpy, and have (8717-8174) divided by this change rate to have the temperature decrease with respect to 300K. Final result is 281K, 19K decrease.
(b)Again the temperature must drop. Between 100K and 200K (at 1 bar) the enthalpy changes by 29.4J/K. Our initial enthalpy, 4442J, is less than 5800 J (the value at 1 bar and 200K) by 1358J, so the temperature must drop by 1358/29.4=46K. The final temperature is therefore 154K.
(c)Starting at 100K, the enthalpy is -1946J. This value like between the enthalpies of the saturated liquid and saturated gas at 1 bar, so we end up with a mixture of liquid and gas at 77K. To find the fraction x that ends up as liquid, just do another interpolation:

$$-3407x + 2161(1-x) = -1946 \Rightarrow x = \frac{2161 + 1946}{2161 + 3407} = 0.74.$$

(d)The highest temperature at which some (infinitesimal)liquefaction takes place would be the temperature at which the initial enthalpy is 2161J. Now at 100 bars, between 100K and 200K, the enthalpy rises by 63.9J/K. But 2161 is greater than -1946 by 4107, so the initial temperature is greater than 100K by

4107/63.9=64.3K, that is, 164K.
(e)At 600K, the enthalpy at 1 bar is slightly less than at 100 bar, so the constant-enthalpy throttling process would have to result in an increase in temperature.