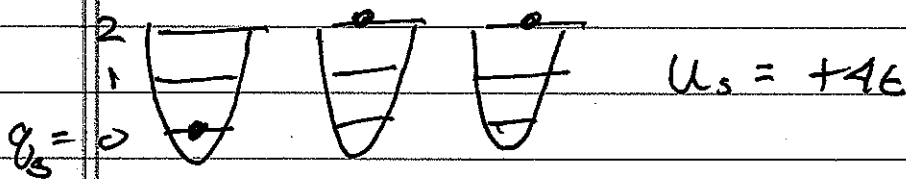


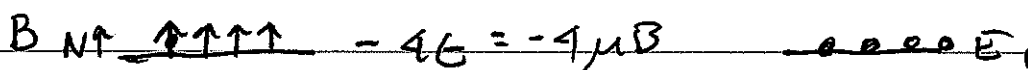
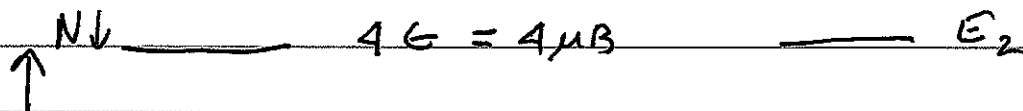
SOLUTIONS FOR PRACTICE MIDTERM 1 ^①

Problem 1

(a) $N_S = 3$ $U_S = +4E$ ($g_S = 4$)



(b) $U_{pi} = -4E$ 1



(c) $\Omega_{system} = \Omega_{pm} \Omega_S$

Elast. OSC: $N_S = 3$ $g_S = 4$

$$\Omega_S = \frac{[g_S + (N_S - 1)]!}{g_S! (N_S - 1)!} = \frac{[4 + (3 - 1)]!}{4! (3 - 1)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 4 \cdot 3 \cdot 2} = 15$$

* Ω_{pm} $N = 4$ $N \uparrow = 4$ $N \downarrow = 0$

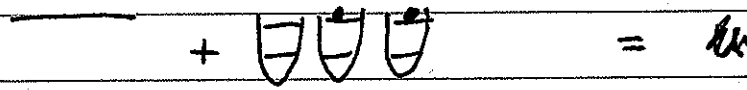
$\Omega_{pm} = 1$ (only one way)

verify: $\Omega_{pm} = \frac{N!}{(N \uparrow)! (N \downarrow)!} = \frac{4!}{(4!) (0)!} = 1 \checkmark$

$$\Omega_{\text{system}} = 15 \times 1$$

(d) Let PM, EO exchange energy \rightarrow at equilibrium will be in state of max Ω_{sys} .
 [There are 3 distinct macropart'ns to consider]

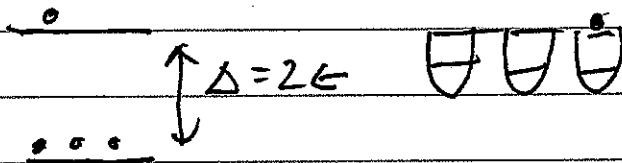
Macropartition # 1



$$U_{\text{PM}} = -4\epsilon \quad U_{\text{EO}} = 4\epsilon \quad U = 0$$

$$\Omega_{\text{PM}} = 1 \quad \Omega_{\text{EO}} = 15 \quad \Omega_{\text{sys}} = 15$$

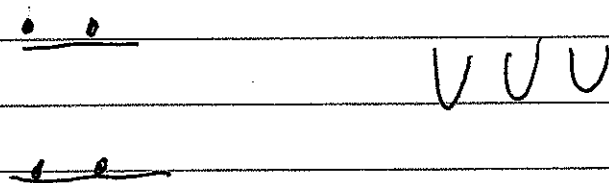
Macropart'n # 2



$$U_{\text{PM}} = -3\epsilon + \epsilon \quad U_{\text{EO}} = 2\epsilon \quad U = 0$$

$$\Omega_{\text{PM}} = \frac{4!}{3!1!} = 4 \quad \Omega_{\text{EO}} = \frac{(2+(3-1))!}{2!2!} = 6 \quad \Omega_{\text{sys}} = 24$$

Macropartition # 3

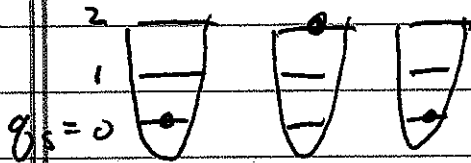


$$U_{\text{PM}} = -2\epsilon + 2\epsilon \quad U_{\text{EO}} = 0 \quad U = 0$$

$$\Omega_{\text{PM}} = \frac{4!}{2!2!} = 6 \quad \Omega_{\text{EO}} = 1 \quad \Omega_{\text{sys}} = 6 \times 1$$

> most probable ^{system} state is macropartition #2

(d) For this state we have $U_{EO} = 2\epsilon$



this is a possible microstate of EO.

(e) Here is corresponding microstate of PM

o $E_2 = +N\mu_B$

$U_p = -2\epsilon$

o o o $E_1 = -N\mu_B$

Problem 2 0.25 moles of ideal monatomic gas
 $T = 0^\circ\text{C} = 273\text{K}$ $P = 1\text{ bar}$
 $\phantom{T = 0^\circ\text{C} = 273\text{K}}$ $= 10^5\text{ Pa}$

(a) Initial state P, V, T
 $\checkmark \quad ? \quad \checkmark \quad \checkmark$
 $P_i V_i = n R T_i$

$$(V_i) = \frac{n R T_i}{P_i} = \frac{(0.25) \left(8.3 \frac{\text{J}}{\text{mol} \cdot \text{K}} \right) (273\text{K})}{10^5 \text{ N/m}^2}$$

$$= 5.7 \times 10^{-3} \text{ m}^3$$

(b) $W = -P_i \Delta V = -P_i (V_f - V_i)$ $W = -900\text{ J}$

$$-\frac{W}{P_i} = V_f - V_i \quad V_f = V_i - \frac{W}{P_i}$$

$$= 5.7 \times 10^{-3} \text{ m}^3 - \frac{(-900\text{ J})}{10^5 \text{ N/m}^2}$$

$$= 0.0147 \text{ m}^3$$

So: $T_f = \frac{P_f V_f}{n R} = \frac{(10^5 \text{ N/m}^2) (0.0147 \text{ m}^3)}{(0.25) (8.3 \text{ J/molK})}$

$$= 707\text{ K}$$

(c) $\Delta U =$ change in internal energy

$$\Delta U = \delta Q + W = 3400\text{ J} - 900\text{ J} = 2500\text{ J}$$

(d) Change in entropy ΔS

Ideal gas \rightarrow Sackur-Tetrode equation

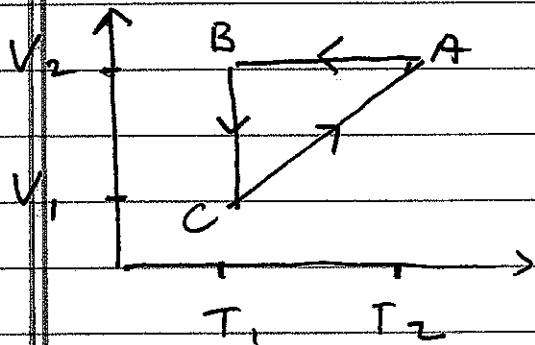
$$S = nR \ln V + \frac{3}{2} nR \ln u$$

$$\Delta S = nR \ln \frac{V_f}{V_i} + \frac{3}{2} nR \ln \frac{u_f}{u_i}$$

$$\frac{\Delta S}{nR} = \ln \left(\frac{0.0147}{0.0057} \right) + \frac{3}{2} \ln \left(\frac{707K}{273K} \right)$$

$$\Delta S = (0.25) \left(8.314 \frac{J}{mol K} \right) [0.948 + 0.9516]$$
$$= 4.9 J/K.$$

Problem 3 one mole of monatomic gas



$$\frac{V_2}{T_2} = \frac{V_1}{T_1}$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

Sackur-Tetrode: ($n = \text{constant} = 1$)

$$S = nR \ln V + \frac{3}{2} nR \ln U$$

$$A-B: \Delta S = \frac{3}{2} nR \ln \frac{U_f}{U_i} \quad U_f = \frac{3}{2} nRT_1$$

$$U_i = \frac{3}{2} nRT_2$$

$$= \frac{3}{2} R \ln \frac{T_1}{T_2} = -\frac{3}{2} R \ln \frac{T_2}{T_1} < 0$$

$$B-C: \Delta S = nR \ln \frac{V_f}{V_i} = R \ln \frac{V_1}{V_2} = R \ln \frac{T_1}{T_2}$$

$$< 0$$

$$C-A: \Delta S = nR \ln \frac{V_2}{V_1} + \frac{3}{2} nR \ln \frac{T_2}{T_1}$$

$$= R \ln \frac{T_2}{T_1} + \frac{3}{2} R \ln \frac{T_2}{T_1} = \frac{5}{2} R \ln \frac{T_2}{T_1}$$

entire cycle $\Delta S = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA}$

Reversible!

$$= -\frac{3}{2} R \ln \frac{T_2}{T_1} - R \ln \frac{T_2}{T_1} + \frac{5}{2} R \ln \frac{T_2}{T_1}$$

Problem 4 0.5 kg of metal at 80°C into large pool of water at 20°C

$$C_p = 100 \text{ J/kg} \cdot \text{K} \quad C = m C_p$$

$$a) \Delta S_{\text{metal}} = \int_{T_i}^{T_f} \frac{\delta Q}{T} = \int_{T_i}^{T_f} \frac{C}{T} dT$$

$$= (100 \text{ J/kg} \cdot \text{K}) (0.5 \text{ kg}) \int_{(80+273)\text{K}}^{293\text{K}} dT/T$$

$$= 50 \text{ J/K} \ln \frac{293\text{K}}{353\text{K}} = -9.3 \text{ J/K}$$

[metal loses entropy b/c it cooled down]

$$b) \Delta S_{\text{water}} = \frac{\text{heat gained from metal}}{T}$$

$$= - \frac{C(T_f - T_i)}{T_f} = \left(\frac{-50 \text{ J}}{\text{mol} \cdot \text{K}} \right) \left(\frac{293\text{K} - 353\text{K}}{293\text{K}} \right)$$

$$= +10.24 \text{ J/K}$$

$$\Delta S_{\text{TOTAL}} = \Delta S_{\text{metal}} + \Delta S_{\text{water}} = (-9.3 + 10.24) \text{ J/mol} \cdot \text{K}$$

$$= +0.94 \text{ J/K} \quad (\text{increase})$$

NOT REVERSIBLE

Problem 5 Rapid compression of air

$$P_i = 105 \text{ Pa} = (105) \cdot 10^5 \text{ N/m}^2$$

$$T_i = \text{~~220~~ } 220 \text{ C} = (220 + 273) \text{ K} = 493 \text{ K}$$

$$V_f = \frac{1}{4} V_i \quad \text{Air: ideal diatomic gas } f = 5$$

Rapid: no time to exchange heat with environment $\int Q = 0 = T ds$
 $ds = 0$ (isentropic)

For isentropic process

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad \gamma = \frac{f+2}{f} = \frac{7}{5}$$

also have for ideal gas (fixed n)

$$\frac{P_1 V_1}{T_1} = nR = \frac{P_2 V_2}{T_2} \quad \frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1}$$

$$\text{So: } \frac{P_2}{P_1} = \frac{V_1^\gamma}{V_2^\gamma} \quad \frac{T_2}{T_1} = \left(\frac{V_1^\gamma}{V_2^\gamma} \right) \left(\frac{V_2}{V_1} \right) \\ = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} = \left[\frac{V_1}{\frac{1}{4} V_1} \right]^{\gamma-1} = 4^{\gamma-1} = 4^{\frac{7}{5}-1} = 1.74$$

$$T_2 = (493 \text{ K})(1.74) = 858 \text{ K}$$