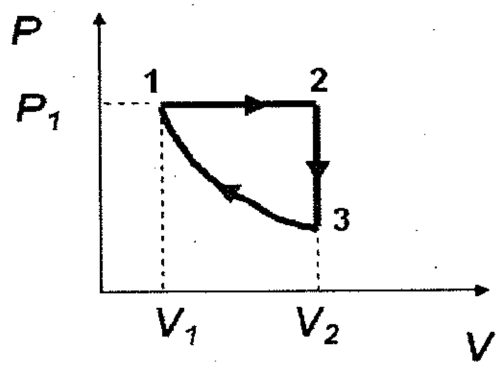
Problem 1 (35 Points)



One mole of a monatomic ideal gas goes through a quasistatic three stage cycle (1-2,2-3,3-1) as shown in the figure at the left. Process 3-1 is adiabatic. The volumes V_1 and V_2 and the pressure P_1 are given. All answers should be given in terms of P_1, V_1 and V_2.

	Work W	Sign W (+/-/0)	Entropy S	Sign ΔS (+/-/0)
1-2	$-P_1(V_2-V_1)$	$-$	$5/2 R \ln(V_2/V_1)$	+
2-3		0	$-5/2 R \ln(V_2/V_1)$	$-$
3-1	$\frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$	+	0	0

(a) (15 Points) Calculate the work W done on the gas during each step of the cycle (1-2,2-3,3-1). Add your answers to the table above, being careful to indicate whether work is positive, negative, or 0.

W_{12} const P $\quad W_{12} = -\overset{+}{P_1}\,(\overset{+}{V_2-V_1}) < 0$

W_{23} const V $\quad W = 0$

W_{31} isentrope = quasistatic + adiabatic $\quad P_3V_3^{\gamma} = P_1V_1^{\gamma} = PV^{\gamma}$

$$W = -\int_{V_2}^{V_1} p(V)\,dV = -\int_{V_2}^{V_1} \frac{P_1V_1^{\gamma}}{V^{\gamma}}\,dV \qquad \gamma = 5/3$$

$$W = -P_1V_1^{\gamma}\frac{1}{1-\gamma}\left[\frac{1}{V^{\gamma-1}}\right]_{V_2}^{V_1} =$$

$$= \frac{3}{2}P_1V_1^{5/3}\left[\frac{1}{V_1^{2/3}} - \frac{1}{V_2^{2/3}}\right] = \frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

Sackur-Tetrode: $S = nR\ln V + \frac{3}{2}nR\ln U + f(n)$

(b) (15 Points) Calculate the entropy change ΔS for each of the three stages (1-2,2-3,3-1). Add your answers to the table above, being sure to indicate whether each entropy change is positive, negative, or zero.

$$\Delta S_{12} = nR\ln\left(\frac{V_f}{V_i}\right) + \frac{3}{2}nR\ln\frac{U_f}{U_i} \qquad \frac{U_f}{U_i} = \frac{T_f}{T_i} \qquad \frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f}$$

$$= R\ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2}R\ln\frac{V_2}{V_1} = \frac{5}{2}R\ln\frac{V_2}{V_1} > 0$$

$$\Delta S_{23} = \frac{3}{2}R\ln\frac{U_f}{U_i} \qquad \frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f} \qquad \frac{T_f}{T_i} = \frac{P_3V_2}{P_1V_2} = \frac{P_3}{P_1}$$

along an isentrope we have $P_3V_3^\gamma = P_1V_1^\gamma$

$$\text{so } \frac{P_3}{P_1} = \left(\frac{V_1}{V_3}\right)^\gamma = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$\Delta S_{23} = \frac{3}{2}R\ln\frac{P_3}{P_1} = \frac{3}{2}R\ln\left(\frac{V_1}{V_2}\right)^\gamma = \frac{3}{2}\gamma R\ln\frac{V_1}{V_2}$$

$$= \left(\frac{3}{2}\right)\left(\frac{5}{3}\right)R\ln\frac{V_1}{V_2} = -\frac{5}{2}R\ln\left(\frac{V_2}{V_1}\right) < 0$$

$\Delta S_{31} = 0$ (isentrope)

(c)(5 Points) Is this cycle reversible? Why or why not?

Reversible since $\Delta S_{cycle} = \Delta S_{12} + \Delta S_{23} + \Delta S_{31}$

$= 0$

Problem 2 (40 Points)

You have two 2-level paramagnets, both of which are in an applied field B at the same temperature T. Here is the direction of B: ↑ Paramagnet A has 3 moments, each with magnitude μ, while paramagnet B has 2 moments, both with magnitude μ.

a)(2 points) What are the minimum and maximum values that energies for the two paramagnets (U_A and U_B) can have?

Paramagnet A: $-3\mu B$ < U_A < $+3\mu B$

Paramagnet B: $-2\mu B$ < U_B < $+2\mu B$

b) (4 points) Make a diagram which shows all of the different macrostates available to Paramagnet A, indicating the total energy U_A and the multiplicity Ω_A for the macrostate. Include in your diagram a picture of a possible microstate which is associated with each macrostate, making sure to indicate the directions of the moments relative to the field B.

↑B

$N\downarrow$	—	↓	↓↓	↓↓↓
$N\uparrow$	↑↑↑	↑↑	↑	—
	$U_A = -3\mu B$	$U_A = -\mu B$	$U_A = +\mu B$	$U_A = +3\mu B$
	$\Omega_A = 1$	$\Omega_A = \frac{3!}{2!\,1!} = 3$	$\Omega_A = 3$	$\Omega_A = 1$

c) (4 points) Make a diagram which shows all of the different macrostates available to Paramagnet A, indicating the total energy U_A and the multiplicity Ω_A for the macrostate. Include in your diagram a picture of a possible microstate which is associated with each macrostate, making sure to indicate the directions of the moments relative to the field B.

—	↓	↓↓
↑↑	↑	—
$U_B = -2\mu B$	$U_B = 0$	$U_B = +2\mu B$
$\Omega_B = 1$	$\Omega_B = 2$	$\Omega_B = 1$

d)(24 Points) Construct a table showing the possible energies **U** for the system obtained when Paramagnets A and B are combined. For each, indicate the total energy **U** as well as the energies U_A and U_B, and calculate the multiplicities Ω_A and Ω_B of Paramagnets A and B which contribute to each macropartition. Finally, indicate the total multiplicity Ω_{AB} for the macropartition.

Construct your table here, using the next page for any supporting calculations.

[in units of μB]

U	U_A	U_B	Ω_A	Ω_B	Ω_{AB}
-5	-3	-2	1	1	1
-3	-3	0	1	2	5
	-1	-2	3	1	
-1	-1	0	3	2	10
	-3	$+2$	1	1	
	$+1$	-2	3	1	
$+1$	1	0	3	2	10
	3	-2	1	1	
	-1	2	3	1	
$+3$	3	0	1	2	5
	1	2	3	1	
$+5$	3	2	1	1	1

e) (3 Points) What is the total number of microstates for this system?

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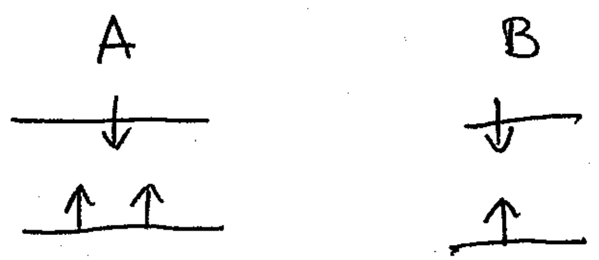
f) (3 Points) Indicate which macropartition or macropartitions in your table has/have the most multiplicity. Make a picture of a microstate including both paramagnets A and B which belongs to this/these most probable macropartition(s).

Macropartitions have $U = \pm \mu B$ have largest multiplicity (18)

example microstate : $U = -\mu B$



$U_A = -\mu B$ $\quad\quad$ $\mu_B = 0$

Problem 3 30 Points

(7 Points) (a)You have a mole of an ideal monatomic gas at 300 K which undergoes an isentropic expansion from an initial volume V_i= 1 liter to a final volume V_f=2 liters. What is the final temperature T_f of the gas?

Isentropic expansion $P_i V_i^{\gamma} = P_f V_f^{\gamma}$ $\gamma = \frac{5}{3}$

but for ideal gas $\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$ $\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i}$

$$\frac{P_f}{P_i} = \left(\frac{V_i}{V_f}\right)^{\gamma} \Rightarrow \frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma}\left(\frac{V_f}{V_i}\right) = \left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

$$\frac{T_f}{T_i} = \left(\frac{1\text{ liter}}{2\text{ liters}}\right)^{\frac{5}{3}-1} = \left(\frac{1}{2}\right)^{2/3} = 0.63$$

$$T_f = (300K)(0.63) = 189K.$$

(b) You have a mole of an ideal monatomic gas at 300 K which undergoes an isothermal and quasistatic expansion from an initial volume V_i= 1 liter to a final volume V_f=2 liters.

i. (5 Points) What is the magnitude of the heat lost or gained by the gas during this process?

Since $T_i = T_f$, we have $U_f = U_i$ $(\Delta U = 0)$

1^{st} Law: $\Delta U = \delta Q + W = 0$ $\quad \delta Q = -W$

On an isotherm, we have $PV = nRT$

$$P(V) = \frac{nRT}{V}$$

$$W = -\int_{V_i}^{V_f} P(V)\,dV = -nRT \int_{V_i}^{V_f} V\,dV$$

$$= -RT \ln \frac{V_f}{V_i} = -RT \ln 2 = -\left(8.3 \frac{J}{mol\,K}\right)(300K)\ln 2$$

$$= -1726\ J \qquad |W| = 1726\ J$$

ii. (3 Points) Which statement best describes the heat flow during this process (circle one) ?

The gas transfers heat to the environment.

The environment transfers heat to the gas. otherwise it would cool down

No heat is exchanged between the gas and the environment.

c) You initially have a mole of an ideal monatomic gas at 300 K in a sealed container with a volume of 1 liter. A second mole of the same gas is added to the same container, using an isothermal and quasistatic process.

i. (5 Points) How much work W was done on or by the ideal gas?

~~[illegible]~~ Since $\Delta V = 0$ $\quad W = 0$

ii. (5 Points) What is the overall change in the ideal gas energy U ?

for ideal gas $\quad U = f N \frac{k_B T}{2}$

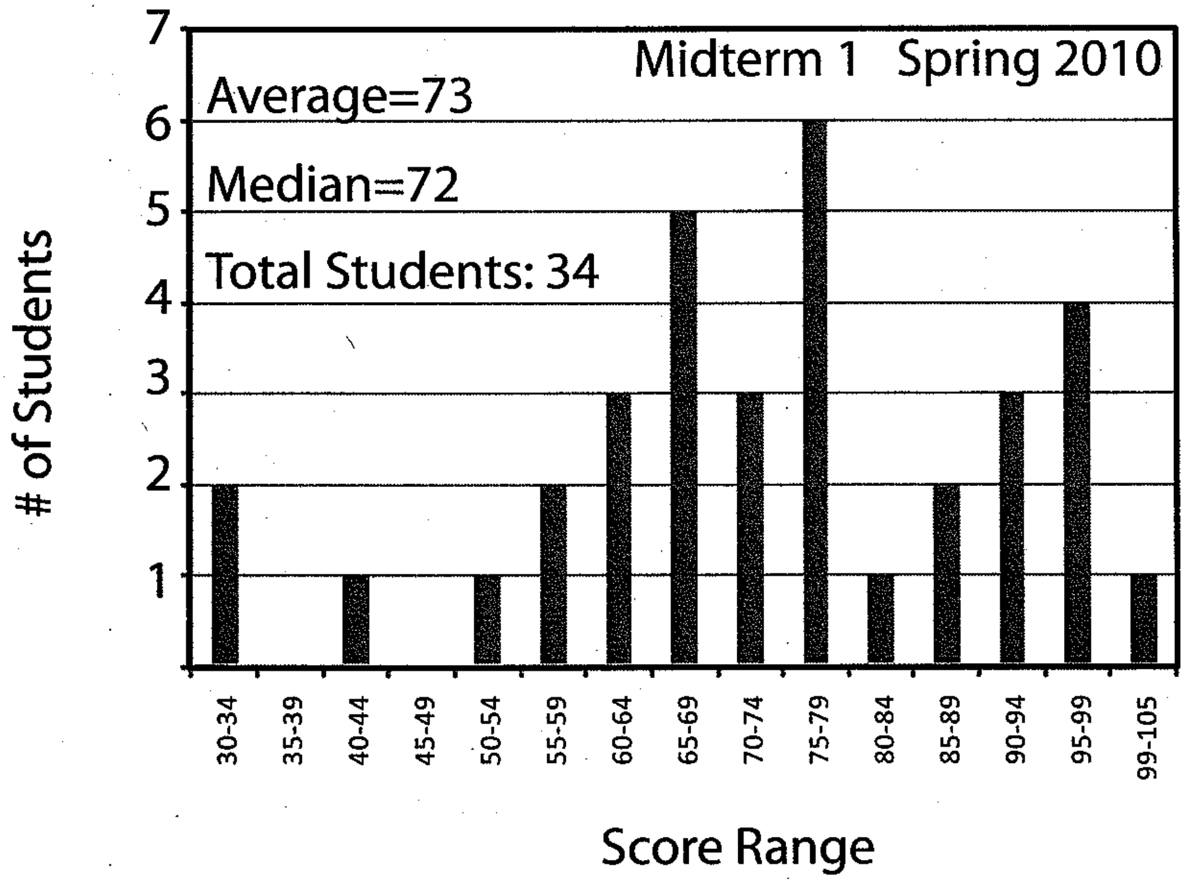
$$U_i = \frac{3}{2} \left(1.38\times10^{-23} \frac{J}{K}\right)(300K)(6.02\times10^{23} \text{ molecules})$$

$$= 3738 \text{ J}$$

$$U_f = 2U_i = 7477 \text{ J} \qquad \Delta U = 3738 \text{ J}$$

iii. (5 Points) What is the value of the chemical potential μ of the ideal gas in this system?

$$\mu \sim \frac{\Delta U}{\Delta N} = \frac{3738 \text{ J}}{6.02\times10^{23} \text{ ptcls}} = 6.23\times10^{-21} \frac{J}{\text{molec.}}$$


Midterm 1 Spring 2010
Average=73
Median=72
Total Students: 34
# of Students
7
6
5
4
3
2
1
30-34
35-39
40-44
45-49
50-54
55-59
60-64
65-69
70-74
75-79
80-84
85-89
90-94
95-99
99-105
Score Range